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TRANSFORMER COUPLING OF INDUCTIVE AND RESISTIVE LOADS TO A MAGNETIC CUMULATION GENERATOR

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Magnetic cumulation (or explosive magnetic) generators are promising as high-power pulsed electrical energy sources [1-3]. When the load is connected directly into the circuit of the magnetic cumulation generator (MCG), the latter can operate efficiently only if restrictions are imposed on the inductance and resistance of the load, whereas in many applications the load parameters substantially exceed the inductance and resistance of the MCG, and the required time for energy input to the load may differ substantially from the general working time. One of the ways of matching the MCG parameters to the load is to use a stepup transformer [1]. Some designs of MCG with transformers have been described [3-7], with discussions of the matching of MCG to resistive and inductive loads. Some applications of transformer MCG in physics research have been discussed in [8-10].

Here we consider forms of transformer output from MCG to inductive and resistive loads. An electrotechnical model is convenient for engineering calculations on transformer MCG, as supplemented with the experimental fact that there is an energy-optimal finite inductance for the generator.

1. In the electrotechnical model, the operation of the MCG is described by a series RL circuit with variable inductance L and resistance R, which formally includes all the losses of magnetic flux Φ. Then $I = \varphi\Phi_0/L$, where I is the current in the generator and $\varphi = \exp\left(-\int_0^t \frac{R}{L}\,dt\right)$, while the subscripts 0 and f denote the values of quantities, respectively, at the start and end of the operation of the MCG. If $|dL/dt| > R$, I increases, while the magnetic energy W increases if $|dL/dt| > 2R$. If $L_f \to 0$ when these conditions are met, then $I_f \to \infty$, which lacks physical meaning, and in that case the problem falls outside the framework of the electrotechnical model. In practice there is some minimum permissible value L_f for each generator.

Figure 1 shows the equivalent electrotechnical scheme for an MCG with a transformer working into a resistance R_l and inductance L_l with switch K closed and constant L_2 and R_2, which is described by the system of equations

$$d(L_1I_1)/dt + R_1I_1 + L_{12}dI_2/dt = 0; \tag{1.1}$$
$$L_2dI_2/dt + R_2I_2 + L_{12}dI_1/dt = 0, \tag{1.2}$$

where $L_1 = L_g + L_{1t}$, L_g is the working inductance of the MCG, L_{1t} is the inductance of the primary winding of the transformer, which includes L_c, the inductance of the current lead from the MCG to the transformer; $L_2 = L_l + L_{2t}$, L_{2t} is the inductance of the secondary winding in the transformer; $L_{12} = k(L_{1t}L_{2t})^{-1/2}$, L_{12} is the mutual inductance, k is the transformer coupling coefficient on the basis of L_c, while R_1 and R_2 are the circuit resistances, and R_l appears in R_2.

If $R_2 = 0$ we have from (1.1) and (1.2) that

$$I_1 = \varphi_e\Phi_0/L_e,\quad I_2 = -I_1L_{12}/L_2 + I_{10}L_{12}/L_0 + I_{20},$$

where

$$L_e = L_1 - L_{12}^2/L_2;\quad \Phi_0 = I_{10}\left(L_0 - L_{12}^2/L_2\right);\quad \varphi_e = \exp\left(-\int_0^t \frac{R_1}{L_e}\,dt\right);$$

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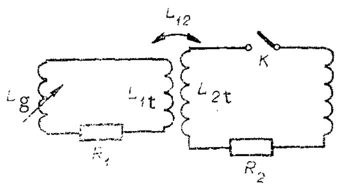

Fig. 1

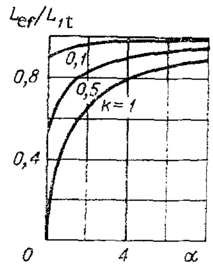

Fig. 2

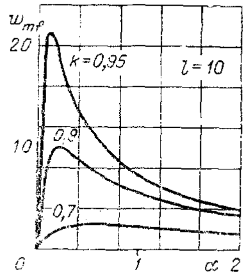

Fig. 3

where L_0, I_{10}, I_{20} are the initial values of L_1, I_1, I_2; in that case the effect of the secondary circuit is to reduce L_1 to L_e, and the MCG is loaded not by $L_{1f} = L_{1t}$ but by L_{ef}. If the optimum lies outside the framework of the model, this enables one to choose L_{ef} by using experimental data or the particular MCG as appropriate to the final inductance. The magnetic energy in the load is $W_m = \psi W$, where W is the total magnetic energy in the MCG and $\psi = L_l L_{12}^2/[L_2(L_1L_2 - L_{12}^2)]$.

For $k = 1$ we have $\psi_f = 1/(1+\alpha) < 1$ if $\alpha = L_l/L_{2t} \neq 0$, i.e., even a transformer with ideal coupling does not allow one to transfer all the generator energy into an inductive load, and $\psi_f \to 1$ for $\alpha \to 0$, but then $L_{ef} = L_{1t}(1-\psi_f) \to 0$ and may become less than the optimum value of the final inductance of the MCG. For $k \neq 1$ we have $L_{ef} = L_{1t}[1 - k^2/(1+\alpha)]$, and Fig. 2 shows the dependence of L_{ef}/L_{1t} on α for various k. The value of ψ_f is maximal for $L_{ef}/L_{1t} = (1-k^2)^{1/2}$.

If also $R_1 = 0$, then the final energy of the MCG and load will be determined by the dimensionless parameters k, α, and l:

$$w_f = \frac{l(1+\alpha)-k^2}{1+\alpha-k^2}, \quad w_{mf} = \frac{\alpha k^2}{(1+\alpha-k^2)^2}\left(l - \frac{k^2}{1+\alpha}\right),$$

where $l = L_0/L_{1t}$; $w_f = W_f/W_0$; $w_{mf} = W_{mf}/W_0$. Figure 3 shows w_{mf} as a function of α for various k for $l = 10$. At the maximum w_{mf} we have $L_{ef}/L_{1t} = (1-k^2)/(1-k^2/2)$, $\alpha = 1 - k^2$, and for $R_1 \neq 0$ the position of the maximum is determined by the character of the magnetic-flux decay. For example, in the case $L_1 = L_0(1 - at)$, where a is a positive constant with the dimensions of $\sec^{-1}$, which is characteristic of coaxial MCG and also of MCG with spirals of constant pitch and R_1, we have that w_{mf} has a maximum for $\alpha = [\sqrt{\mu^2k^4 + 4(1-k^2)} - \mu k^2]/2$, where $\mu = 1 + 2R_1/(dL_1/dt)$, which is true also for an exponentially decreasing inductance $L_1 = L_0e^{-at}$, which approximately describes the inductance law for spirals of variable pitch and constant L_1/R_1.

For the given L_{2t} the necessary α is attained by selecting L_l. On the other hand, in selecting a transformer for a particular value of L_l we can provide the required α by varying L_{1t} or L_{12}, i.e., by choosing the number of turns on the secondary winding. If it is possible to provide a constant value of k by design (which is not always possible), it is necessary to select $L_{12} = k\sqrt[4]{L_{1t}^2L_l^2(1-k^2)}$ for the maximum ψ_f, while for the maximum w_{mf} (for $R_1 = 0$) we must have $L_{12} = k[L_{1t}L_l(1-k^2)]^{1/2}$. The relation between L_{12} and L_{1t} is determined by the transformer design. If, for example, $L_{1t} = L_{12}/(kN)$, where N is the number of turns on the secondary winding, then for given L_{1t} and k it is necessary in order to obtain maximum ψ_f that $N = (L_l/L_{1t})^{1/2}(1-k^2)^{1/4}$, and for the given N and k it is necessary to have $L_{1t} = L_l/[N^2(1-k^2)^{1/2}]$.

2. To illustrate the effects of R_2 we consider the solution to (1.1) and (1.2) for $R_1 = 0$, $I_{20} = 0$, i.e., switch K is closed at the start of operation of the MCG. Then (1.1) and (1.2) are solved in quadratures for uniform and exponential laws for L_1. For example, in the second case we can write

$$i_2 = l(1+\alpha)[l(1+\alpha) - k^2e^{at}]^{\nu-1}e^{-a\nu t}\int_1^z\left[\frac{z}{l(1+\alpha)-k^2z}\right]^\nu dz,$$

with the final value

$$i_{2f} = (1+\alpha)(1+\alpha-k^2)^{\nu-1}\int_1^l\left[\frac{z}{l(1+\alpha)-k^2z}\right]^\nu dz,$$

where

$$i_1 = I_1/I_{10};\ i_2 = -I_2L_2/(L_{12}I_{10});\ z = L_0/L_1 = e^{at},$$

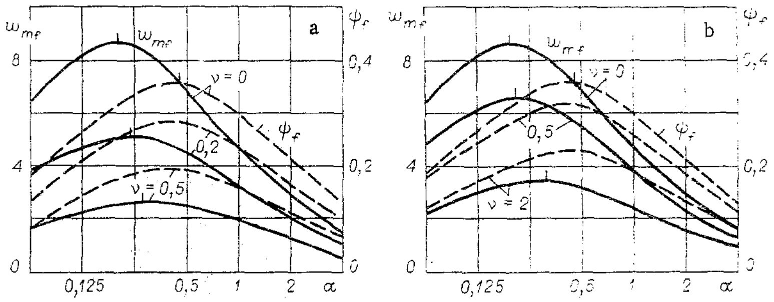

Fig. 4

and $\nu = R_2/(\alpha L_2)$. Analogous expressions may be derived for a uniform law for L_1. The final values for the energy coefficients in both cases are given by

$$w_{mf} = \frac{\alpha k^2 i_{2f}^2}{l(1+\alpha)^2}, \quad \psi_f = \frac{\alpha k^2 i_{2f}^2}{l^2(1+\alpha)^2 + k^2(1+\alpha-k^2)\, i_{2f}^2},$$

$$w_{qf}\ (\text{uniform law}) = \frac{2\nu k^2}{l(1+\alpha)} \int_1^l \frac{i_2^2 dz}{z^2}, \quad w_{qf}\,(\text{exponential}) = \frac{2\nu k^2}{l(1+\alpha)} \int_1^l \frac{i_2^2 dz}{z},$$

where $w_{qf} = W_{qf}/W_0$ and W_{qf} is the energy deposited in R_2 during the operating cycle T of the MCG. The load R_l receives energy $W_{qf}R_l/R_2$, so the problem is determined by the dimensionless parameters k, l, α, ν.

Figure 4 shows w_{mf} and ψ_f as functions of α for various ν for both cases as calculated for $l = 10$ and $k = 0.9$; the two cases differ in the effects of α and ν on the maximum in the curve [a) $L_1 = L_0 e^{-at}$; b) $L_1 = L_0(1 - at)$].

Figure 5 shows w_{qf} as a function of ν for various α for $l = 10$ and $k = 0.95$ [solid lines $L_1 = L_0 e^{-at}$, broken lines $L_1 = L_0(1 - at)$].

If $R_2 \ll (L_2/I_2)dI_2/dt$, (1.1) and (1.2) have an approximate solution

$$I_1 \approx \Phi_0 \varphi_e / L_e, \quad I_2 \approx -L_{12} I_1 / L_2,$$

where $\varphi_e = \exp\left(-\int_0^t \frac{R_e}{L_e}\, dt\right)$; $R_e = R_1 + R_2 L_{12}^2 / L_2^2$.

It is also possible to obtain an analytical solution to (1.1) and (1.2) if $L_1 = L_0/(1 + at)$, $R_1 = 0$, although this inductance law is not very characteristic of MCG.

3. A transformer also enables one to adjust the shape of the current pulse in the load within certain limits. The maximum power is usually developed at the end of the MCG cycle. The method of [7] allows one to sharpen the leading edge of the current without breaking the circuit; initially, the transformer operates with the secondary circuit open (no load), and switch K is closed only at certain time τ after the start of the MCG. Then the length of the leading edge will be $T - \tau$ (if I_2 rises up to the end of the MCG operation). After the switch is closed, for $R_2 = 0$

$$I_1 = \frac{\Phi_0 \varphi_\tau \varphi_e L_{1\tau}}{\varphi_{e\tau} L_{e\tau} L_e}, \quad I_2 = -\frac{\Phi_0 \varphi_\tau L_{12}}{L_{1\tau} L_2} \left(\frac{\varphi_e L_{e\tau}}{\varphi_{e\tau} L_e} - 1\right),$$

where the subscript τ denotes the value of the corresponding quantity at the instant of closure and dI_1/dt increases stepwise on closure by a factor $L_1/L_{e\tau}$.

W_{mf} decreases as $T - \tau$ decreases. We denote by ε the ratio of W_{mf} for the case of a secondary circuit closed and the value of W_{mf} attained when the switch is closed at time τ, which characterizes the degree of use of the MCG energy. Then $\varepsilon = (1 - L_{1t}/L_{1\tau})^2$ for $R_1 = 0$. Then the degree of shortening in the front is $T/(T - \tau) = (l - 1)(\varepsilon^{-1/2} - 1)$ for a uniform law for L_1, while $T/(T - \tau) = -(\ln l)/[\ln(1 - \varepsilon^{-1/2})]$ for an exponential law. If $R_1 \neq 0$, then

$$\varepsilon = \left[\frac{\varphi_\tau}{L_{1\tau}} \left(\frac{L_{e\tau}}{\varphi_{e\tau}} - \frac{L_{ef}}{\varphi_{ef}}\right)\right]^2.$$

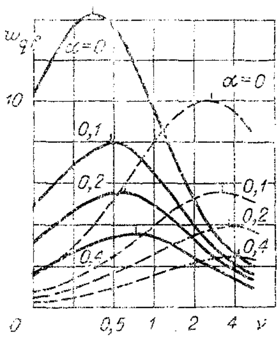

Fig. 5

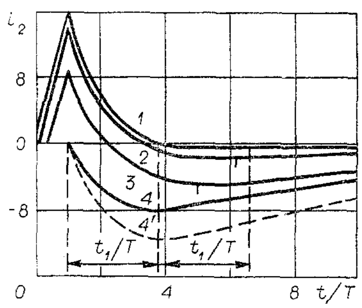

Fig. 6

This method allows one to reduce the leading edge of the current by a factor of 5-10 for an inductive load with an applicable value of ε. It is desirable to have a higher value of Φ_0 for the open-circuit stage in the transformer, since the generator will work with less stress than is the case when the switch is closed. If the W_0 are then identical for the two cases, ε is increased by a factor of $l(1+\alpha)/[l(1+\alpha)-k^2]$, or by $l^2(1+\alpha)^2/[l(1+\alpha)-k^2]^2$ if the I_{10} are the same. In the case of an ohmic load, the sharp end of the current pulse sharpens the power pulse.

If it is necessary to stretch the current pulse in a resistive load over a time exceeding T, one can include a storage inductance in series with R_l to increase L_l, the result being that most of the energy is stored in L_l while the MCG is working and is then deposited in R_l with a relaxation time of L_l/R_l. Here the transformer unit destroyed at the end of the MCG operation is shunted by an additional switch. In this form of MCG, the device essentially charges an inductive store. The higher power of an MCG distinguishes it from low-power supplies for inductive stores and subtantially reduces the resistive loss during charging.

The current decay at the end of MCG operation is described by the standard laws for inductively coupled RL circuits, whose initial conditions are I_{1f} and I_{2f}; if the transformer still functions during the decay time, an additional fraction of the MCG energy can be transferred to the load. If the effects of R_2 are small during operation of the MCG and the secondary circuit is closed, then $i_{2f} = i_{1f}$, and then during current decay

$$i_2 = i_{2f}\left[e^{\lambda_2 t} + \left(e^{\lambda_1 t} - e^{\lambda_2 t}\right)\lambda_1/(\lambda_1 - \lambda_2)\right],$$

where

$$\lambda_{1,2} = [-(\delta_1+\delta_2) \pm \sqrt{(\delta_1-\delta_2)^2 + 4\delta_1\delta_2 k^2/(1+\alpha)}]/[2 - 2k^2/(1+\alpha)];$$
$$\delta_1 = R_1/L_{1t};\ \delta_2 = R_2/L_2.$$

At a time $t_1 = [\ln(\lambda_2/\lambda_1)]/(\lambda_1-\lambda_2)$ after the end of MCG operation we have $i_2 = 0$, which is followed by a change of sign, and the maximum in i_2 of the reverse sign occurs at time $t_2 = 2t_1$, and then i_2 relaxes. The additional energy ΔW_q deposited in R_2 during the current decay is then given by

$$\frac{\Delta W_q}{W_f} = \frac{k^2}{(1+\alpha)(1+\delta_1/\delta_2)},$$

from which the necessity of the condition $\delta_2 \gg \delta_1$ follows. This condition is difficult to meet when it is desired to stretch the current pulse substantially. When α is varied, it is necessary to allow for the effects on the working stage of the MCG, and if here one can neglect R_1, then

$$\frac{W_{qf}+\Delta W_q}{W_0} = \frac{k^2[l(1+\alpha)-k^2]}{(1+\alpha)(1+\alpha-k^2)(1+\delta_1/\delta_2)}.$$

If the secondary circuit is closed at time τ, then $i_{2f} = i_{1f} - i_{1\tau}$ for small R_1 and R_2. Also, i_{2f} decreases as τ increases, and there is an increase in the amplitude of i_2 in the reverse half-wave, with a maximum between t_1 and t_2. There is also an increase in the magnetic flux linked to the secondary circuit at the instant of closure. For $\tau = T$, practically the entire flux of the MCG is involved, and $i_{2f} = 0$, and there is only a reverse current half-wave, whose maximum will occur at t_1, which also defines the length of the current leading edge. Under these conditions the secondary circuit does not influence the operation of the MCG, and for any R_1 and R_2 we have

$$i_2 = \frac{i_{1f}\delta_1(1+\alpha)}{(1+\alpha-k^2)(\lambda_1-\lambda_2)}\left(e^{\lambda_2 t} - e^{\lambda_1 t}\right),$$

$$\frac{\Delta W_q}{W_f} = \frac{k^2}{(1+\alpha)(1+\delta_2/\delta_1)}.$$

If we take $\delta_2 \ll \delta_1$, this reduces W_f, and therefore it is rational to increase δ_1 only after the end of MCG operation, e.g., by breaking the circuit. It is best to reduce δ_2 not by varying α but by reducing R_2 by increasing the quality factor of the secondary winding on the transformer, although this involves increasing the size of the transformer unit.

Figure 6 shows curves for i_2 calculated for a uniform law for L_1 with $l = 10$, $k = 0.9$, $\alpha = 1$, $\delta_1 T = 0.5$, $\delta_2 T = 0.1$ without allowance for R_1 and R_2 in the MCG operation. Curve 1 corresponds to the secondary circuit closed; curve 2 is for $\tau = 0$, and curve 3 is for $\tau = 0.4$ T, while curve 4 is for $\tau = 1$ (flux-trapping mode).

In the last case, the energy L_l is maximal at time t_1 after the closure, and

$$\frac{W_m}{W_f} = \frac{\alpha k \delta_1^2}{(1+\alpha-k^2)^2 \lambda_1^2} \left(\frac{\lambda_2}{\lambda_1}\right)^{2\lambda_2/(\lambda_1-\lambda_2)}.$$

For $\delta_2 \ll \delta_1$

$$i_2 \approx -i_{1f}\left\{1 - \exp\left[-\frac{\delta_1(1+\alpha)t}{1+\alpha-k^2}\right]\right\}, \quad \frac{W_m}{W_f} \approx \frac{\alpha k^2}{(1+\alpha)^2}.$$

If δ_1 is increased rapidly after the end of MCG operation, one can produce for example a current pulse with a sharp leading edge and a prolonged decay if δ_2 is sufficiently small.

It is desirable in flux-trapping mode that L_{1t} should be equal to the optimum final inductance for the given MCG while maintaining the same l, k, and α; this provides working conditions in the generator as with the secondary circuit closed, while the maximum value of i_2 is increased (curve 4' in Fig. 6).

There are also other possible forms of supply circuit for inductive and resistive loads using transformer MCG.

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